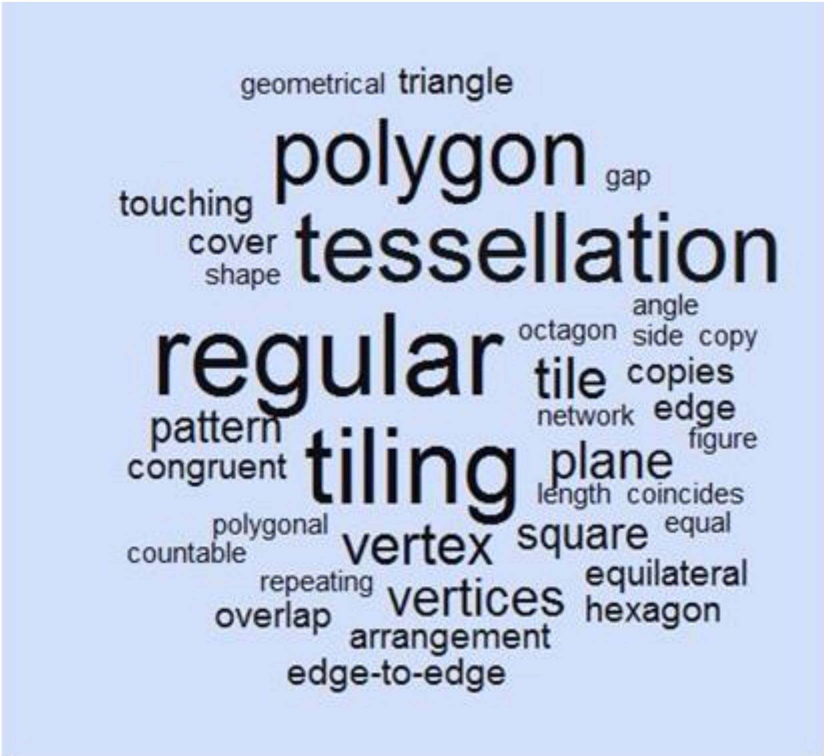
TESSELLATION

"For me it remains an open question whether [this work] pertains to the realm of mathematics or to that of art."

M.C. Escher

Activity 1: Guessing the lesson	
Doc. 1	**Word Cloud**



1) What do you think the lesson will be about?

………

………

2) Which words do you know?

………

………

3) Which words don't you know?

………

………

4) Which words can you add to these?

………

………

Activity 1: Guessing the lesson	
Doc. 2	**Video Questions**

The Mathematical Art of M.C. Escher

Ian Stewart, University of Warwick

BBC 4 (2005)

1) Who was Escher?

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……

2) Where did Escher find inspiration?

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……

3) What is a tessellation?

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4) Why is tessellation about Mathematics?

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……

<u>Activity 1</u>: Correction
Part 1: Word Cloud

The vocabulary required

…………………….: a plane figure bounded by line segments. Some of them: triangle (3), quadrilateral (4), pentagon (5), hexagon (6), heptagon (7), octagon (8), enneagon (9), decagon (10), undecagon (11), dodecagon (12).

………………. **polygon**: a polygon with all sides and all angles equals.

Edge: a line along which two faces or surfaces of a solid meet (edge-to-edge = bord à bord).

……………………: having identical shapes so that all parts correspond, corresponding exactly when superimposed.

………………………: The point at which the sides of an angle intersect.

……………………….: the contour of a thing.

…………………………: a piece of baked clay used to form a roof, to cover a wall.

…………………………: something made with tiles, a tiled surface (pavage, carrelage).

Tessellation: a kind of tiling.

………………………………: a decorative design made up of elements in a regular arrangement (motif).

Arrangement: the act of arranging or being arranged.

To arrange: to place in proper order.

……………: a space between objects or points.

…………………**.**: to cover part of, or something.

<u>Activity 1</u>: Transcript of the video

The Mathematical Art of M.C. Escher
Ian Stewart, University of Warwick
BBC 4 (2005)

« An amazing thing about MC Escher is that he represents the perfect coming together of mathematics and arts. These are two different worlds but in his work they are brought together as one.

Born in the Netherlands in 1898, Mauritius Cornelius Escher had no formal training in mathematics. He began his professional life as a graphic artist: making woodcuts and lithographs. As a young man while visiting the Alhambra in Spain he became fascinated by the geometric decoration of the Moorish tiles. It would be a defining moment for Escher as an artist. From then on he would spend much of his life experimenting with the area of mathematics known as tessellation.

« Tessellation is about regular patterns that divide the plane. That means they fit if they split the plane up into lot of different tiles and those tiles fit together perfectly: they don't overlap and they don't leave any gaps. It may seem that the premise of tiling a plane or surface with regular repeating unit is a very simple idea. But it's absolutely fundamental to mathematics and the reason is that it's about symmetry. »

<u>Activity 2</u>: Focus on language	
<u>Doc. 1</u>	**Text and matching**

a) Read the text "Plane tiling and tessellations" carefully.
b) Some expressions, explained in the text, are illustrated in table n°1.

Fill in table n°2 by associating the appropriate illustration to the expression. You will have to justify your choice by quoting the text.

Plane tiling and tessellations

A tiling of the plane is any countable family of tiles that cover the plane without gaps or overlaps. In this section, our tiles will be polygons.
An edge-to-edge tiling is a polygonal tiling in which each edge of each polygon coincides with an edge of exactly one other polygon, with the vertices of one tile touching only the vertices of others.
An edge-to-edge tiling in which all the tiles are regular polygons (that's to say a shape having sides of the same length and equal angles), is called a tessellation.
A regular tiling is a tessellation in which all the tiles are congruent, and with the same arrangement of polygons at each vertex.
When two or more types of regular polygons are used, with the same pattern at each vertex (the regular polygons should always appear in the same order), we talk about semi-regular tiling.

Table n°1

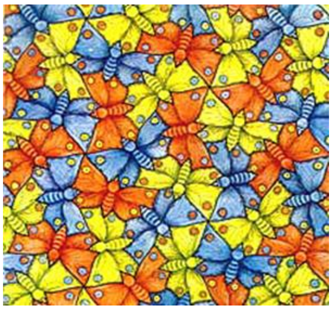
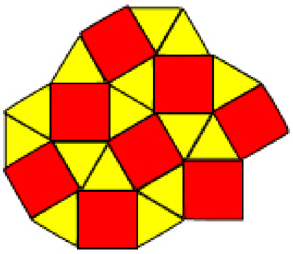
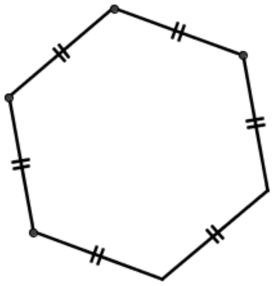
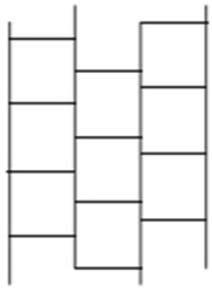
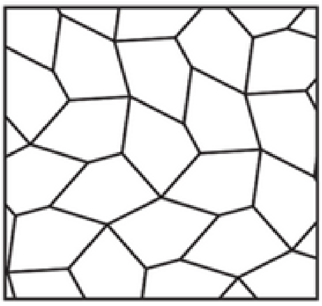
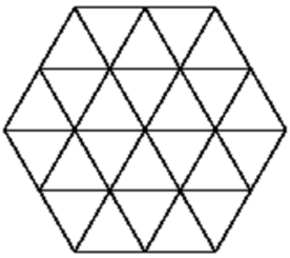
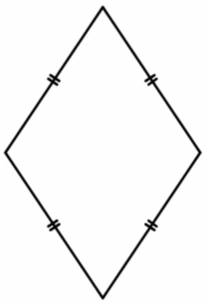
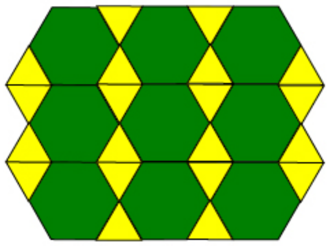
A. 	B. 	C. 	D. 
E. 	F. 	G. 	H. 

Table n°2

Expression from the text	Illustration	Justify your choice by quoting the text
A non-regular polygon.		
A regular polygon.		
A tiling which is not a polygonal one.		
A polygonal tiling which is not an edge-to-edge one.		
An edge-to-edge polygonal tiling which is not a tessellation.		
A tessellation that is not a regular tiling.		
A regular tiling.		
A tessellation that is not a semi-regular tiling.		
A semi-regular tiling.		

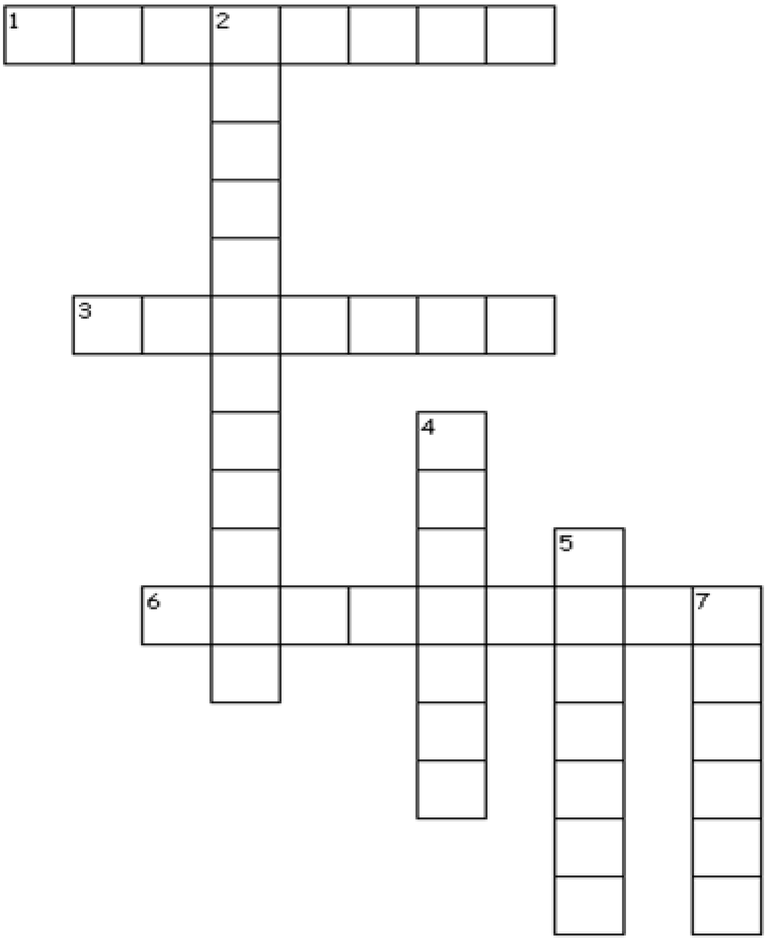
Activity 2: Focus on language	
Doc. 2	**Crossword**

Across

1. The points where lines meet to form an angle.
3. Any 2D shape with straight sides.
6. Describes a shape in mathematics that has the same shape and size as another.

Down

2. Pattern resulting from the arrangement of regular polygons to cover a plane without any gaps.
4. To cover something partly by going over its edge; to cross each other.
5. Squares and equilateral triangles are, but not rectangles.
7. … was a favorite means of expression for the Dutch artist M. C. Escher (1898-1972).



http://puzzlemaker.discoveryeducation.com/CrissCrossSetupForm.asp

Activity 3: Writing frame	
Doc.1	Report describing the different types of regular tiling.

a) Work in pairs.
b) Agree with the definition of a regular tiling.
c) Use the different sets of regular polygons to find out the different types of regular tiling, and an example of a regular polygon which can't give a regular tiling (try to explain why it doesn't work).
d) Sum up your results on a poster, following the guideline below. You will have to present it to another group.

Our names: 1) …………………………………………….

2) …………………………………………….

Definition of a regular tiling:

………
………

Different types of regular tiling:

Illustrations	Polygons involved

Example of a regular polygon which can't give a regular tiling.

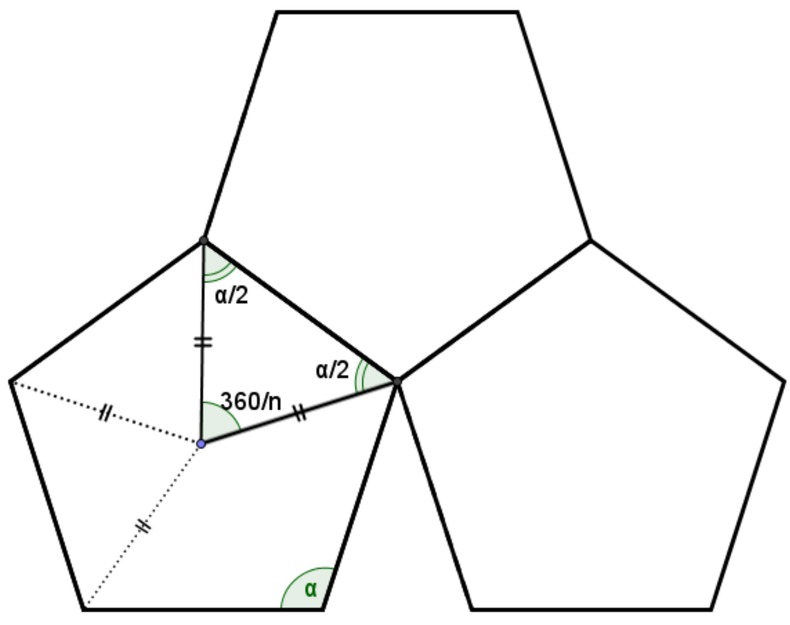
Illustrations	Type of polygon involved	Reason why it doesn't work

Activity 4: A proof	
Doc.1	**Why do the only regular tilings consist of equilateral triangles, squares and hexagons?**

It is easy to check that equilateral triangles, squares and regular hexagons can produce regular tilings.
We want to show that only those three regular polygons (that's to say regular polygons with 3, 4 or 6 sides) can make up regular tilings.

a) You can find below the beginning of the proof, and a diagram which illustrates the problem and the notations used.

"As shown in the diagram, we consider a regular polygon with n sides $(n \geq 3)$.
Let α denote the interior vertex angle of the regular polygon.
Then we divide our polygon into n isosceles congruent triangles."



b) The eight steps of the proof are written in table 1.
Organise the cards, in order to obtain the proof, and at the same time, complete the sentences from table 1 with the appropriate mathematical expression from table 2.

Activity 4: A proof	
Doc.2	**The eight steps of the proof *(table 1)* and the mathematical expressions *(table 2)*.**

Table 1

Yet the only divisors of 4 are …
However, at each vertex, k congruent polygons (k positive integer), with n sides must meet with no gaps, nor overlaps, so we have $k \times \alpha =$ …
$2+\frac{4}{n-2}$ is a whole number if and only if $n-2$ is a divisor of …
Since the sum of the three angles in a triangle has to be $180°$, we can deduce the following equality: … and therefore $\alpha = \ldots$
Consequently the angles of those congruent isosceles triangles are respectively equal to …
Therefore that gives us only three values for $n-2$, and then three possible values for n: …
Assuming that $k = \ldots$, we are looking for regular polygons for which the ratio $\frac{360}{\alpha}$ is a whole number.
However we have: $\frac{360}{\alpha}=\frac{360}{180-\frac{360}{n}}=\frac{360n}{180n-360}=\frac{2n}{n-2}=2+\frac{4}{n-2}$

Table 2

$180-\frac{360}{n}$	$\frac{\alpha}{2}, \frac{\alpha}{2}$ and $\frac{360}{n}$
$\frac{\alpha}{2}+\frac{\alpha}{2}+\frac{360}{n}=180$	$1, 2$ and 4
4	360
$\frac{360}{\alpha}$	$3, 4$ and 6

<u>Activity 4</u>: A proof **Correction**	
<u>Doc.3</u>	**Why do the only regular tilings consist of equilateral triangles, squares and hexagons?**

- As shown in the diagram, we consider a regular polygon with n sides $(n \geq 3)$.
  Let α **denote** the interior vertex angle of the regular polygon. Then we divide our polygon into n isosceles congruent triangles.

- **Consequently** the angles of those congruent isosceles triangles are respectively equal to ....

- **Since** the sum of the three angles in a triangle has to be $180°$, **we can deduce** the following equality: .... and therefore $\alpha =$ ....

- **However**, at each vertex, k congruent polygons (k positive integer), with n sides must meet with no gaps, nor overlaps, **so** we have $k \times \alpha =$ ....

- **Assuming that** $k =$...................., we are looking for regular polygons for which the ratio $\frac{360}{\alpha}$ is a whole number.

- **However** we have: $\frac{360}{\alpha} = \frac{360}{180 - \frac{360}{n}} = \frac{360n}{180n - 360} = \frac{2n}{n-2} = 2 + \frac{4}{n-2}$

- $2 + \frac{4}{n-2}$ is a whole number **if and only if** $n-2$ is a divisor of ....

- **Yet** the only divisors of 4 are ....

- **Therefore** that gives us only three values for $n-2$, and then three possible values for n :...........................

- **We can conclude that** the only regular tilings are made up with regular polygons with 3, 4 and 6 sides that is to say equilateral triangles, squares and hexagons.